Nature's zoo of elementary particles is not a random mishmash; it has striking patterns and interrelationships that can be depicted on a diagram corresponding to one of the most intricate geometric objects known to mathematicians, called E8.
Modem physics began with a sweeping unification: in 1687 Isaac Newton showed that the existing jumble of disparate theories describing everything from planetary motion to tides to pendulums were all aspects of a universal law of gravitation. Unification has played a central role in physics ever since. In the middle of the 19th century James Clerk Maxwell found that electricity and magnetism were two facets of electromagnetism. One hundred years later electromagnetism was unified with the weak nuclear force governing radioactivity, in what physicists call the electroweak theory.

This quest for unification is driven by practical, philosophical and aesthetic considerations. When successful, merging theories clarifies our understanding of the universe and leads us to discover things we might otherwise never have suspected. Much of the activity in experimental particle physics today, at accelerators such as the Large Hadron Collider at CERN near Geneva, involves a search for novel phenomena predicted by the theoretical physics paper of the year. Outlets from the New Yorker to Outside magazine were drawn to the story, partly on account of his surfer lifestyle. Lisi and others have continued to develop the theory. Most physicists think reconciling Einstein’s general theory of relativity with quantum theory will require a radical shift in our conception of reality. Lisi, in contrast, argues that the geometric framework of modern quantum physics can be extended to incorporate Einstein’s theory, leading to a long-sought unification of physics. Even if Lisi turns out to be wrong, the E8 theory he has pioneered showcases striking patterns in particle physics that any unified theory will need to explain.
unified electroweak theory. In addition to predicting new physical effects, a unified theory provides a more aesthetically satisfying picture of how our universe operates. Many physicists share an intuition that, at the deepest level, all physical phenomena match the patterns of some beautiful mathematical structure.

The current best theory of nongravitational forces—the electromagnetic, weak and strong nuclear force—was largely completed by the 1970s and has become familiar as the Standard Model of particle physics. Mathematically, the theory describes these forces and particles as the dynamics of elegant geometric objects called Lie groups and fiber bundles. It is, however, somewhat of a patchwork; a separate geometric object governs each force. Over the years physicists have proposed various Grand Unified Theories, or GUTs, in which a single geometric object would explain all these forces, but no one yet knows which, if any, of these theories is true.

And an even deeper unification problem faces today’s physicists. In a fully unified theory, gravity and matter should also combine naturally with the other forces, all as parts of one mathematical structure—a Theory of Everything. Since the 1980s string theory, the dominant research program in theoretical particle physics, has been an attempt to describe gravity and the Standard Model using elaborate constructs of strings and membranes vibrating in many spacetime dimensions.

But string theory is not the only effort. An alternative, loop quantum gravity, uses a more minimal framework, closer to that of the Standard Model [see “Atoms of Space and Time,” by Lee Smolin; SCIENTIFIC AMERICAN, January 2004]. Building on its insights, one of us (Lisi) proposed a new unified theory in 2007. The basic idea is to extend Grand Unified Theories and include gravity as part of a consistent geometric framework. In this unified field theory, called E8 theory, all forces and matter are described as the twisting of a single geometric object.

All new ideas must endure a trial by fire, and this one is no exception. Many physicists are skeptical—and rightly so. The theory remains incomplete. But even in this early stage of development, it unveils some of the beautiful structures in play at the deepest levels of nature, and it makes predictions for new particles that the Large Hadron Collider might find. Although physicists are not yet at the culmination of our centuries-long quest for unity, E8 theory is an important step on that journey.

**EVERY FIBER OF OUR BEING**

To describe E8 theory, we first need to set out the widely accepted geometric principles that govern all known forces and particles. Geometry is the study of shape, but in the case of fundamental physics, you might wonder: shape of what? Plato thought elements such as earth and air were associated with little cubes and octahedra. Similarly, in modern physics, the geometric objects associated with elementary particles are perfect, smooth shapes, existing outside our space yet connected to it. We cannot see these shapes directly, but we see their effects.

The main geometric idea underlying the Standard Model is that every point in our spacetime has shapes attached to it, called fibers, each corresponding to a different kind of particle. You can envision the universe as a Chia Pet (a terra-cotta figureine covered with sprouts). The surface of the figureine is analogous to spacetime, and the sprouts are fibers. The entire geometric object—spacetime and fibers together—is called a fiber bundle. The fibers are not in our space, but over it; they may be thought of as different, internal spaces attached to each point of our spacetime, with shapes corresponding to particles’ properties.

This idea, introduced in 1918 by mathematician Hermann Weyl, is now a well-established principle of physics [see “Fiber Bundles and Quantum Theory,” by Herbert J. Bernstein and Anthony V. Phillips; SCIENTIFIC AMERICAN, July 1981]. Distinct from the speculated undulating spatial dimensions of string theory, these internal space fibers are of fixed shape. Their dynamics arise from how they are attached to four-dimensional spacetime.

The electric and magnetic fields existing everywhere in our space are the result of fibers with the simplest shape: the circle. A circle, called U(1) by physicists, is the simplest example of a Lie group (pronounced “Lee,” after 19th-century Norwegian mathematician Sophus Lie). It has a single symmetry: if we rotate a circle, it remains the same. A small rotation like this is called a generator of the Lie group. Following a generator, just like drawing with a compass, takes us around a circle.

The fiber bundle of electromagnetism consists of circles attached to every point of spacetime [see box on opposite page]. Crucially, each circle can rotate a little relative to its spacetime neighbors. The so-called connection field of a fiber bundle describes how neighboring fibers are related by these symmetry rotations. The electric and magnetic force fields filling spacetime correspond to the curvature of this fiber bundle—geometrically, the electric and magnetic fields are how the circular fibers twist over time and space. An electromagnetic wave is the undulation of circles over spacetime. One quantum of an electromagnetic wave—a photon—is a propagating particle of light.

Each kind of elementary particle corresponds to a different fiber over spacetime; the Chia Pet has many different kinds of sprouts. All the electrons of the world result from the twisting of a single kind of fiber—explaining, among other things, why all electrons are identical. The fibers of electrically charged particles, such as electrons, wrap around the circular fibers of electromagnetism like threads around a screw. How fast a particle’s fiber twists around the circle is equal to its electric charge, determining how the particle responds to the force of electromagnetism.

Because twists must meet around the circle, these charges are integer multiples of some standard unit of electric charge. Of the elementary matter particles, called fermions, electrons have electric charge $\frac{1}{3}$ (one twist), up quarks have electric charge $+\frac{2}{3}$ (two opposite twists), down quarks have electric charge $-\frac{1}{3}$ (one twist), and neutrinos have 0. The antimatter particles, such as positrons and antiquarks, have twists in the opposite direction around the electromagnetic circle, giving them the opposite electric charges.

When particles collide, they may be converted into new types, but the outgoing particles have exactly the same total charge as the incoming ones did. This crucial fact is a consequence of fiber geometry: When any two particles meet, their twists add. In this way, the fiber-bundle picture explains what we know about electromagnetism. The electric charges describe the geometric structure of the combined electromagnetic and matter fiber bundle, determining what interactions are possible between electrically charged particles.

**DIFFERENT CHARGES FOR DIFFERENT FORCES**

Physicists apply these same principles to the weak and strong nuclear forces. Each of these forces has its own kind of charge and its own propagating particles. They are described by more
complicated fibers, made up not just of a single circle but of sets of intersecting circles, interacting with themselves and with matter according to their twists.

The weak force is associated with a three-dimensional Lie group fiber called SU(2). Its shape has three symmetry generators, corresponding to the three weak-force boson particles: $W^+$, $W^-$ and $W^3$—relatives of the photon. Each Lie group is a multidimensional, smooth tangle of intersecting circles twisting around one another. The circles of the $W^+$ and $W^-$ bosons in SU(2) twist oppositely around the $W^3$ circles and so have weak charge, $W_1$ of +1 and −1. Because they have weak charge, these particles interact with one another as well as with matter.

Exactly half of elementary matter particles interact with the weak force, their fibers twisting around the $W^3$ and other circles of SU(2). Fermions come in two varieties, related to how their spin aligns with their momentum: left-handed and right-handed. Only the left-handed fermions have weak charges, with the left-handed up quark and neutrino having weak charge $+\frac{1}{2}$ and the left-handed down quark and electron having weak charge $-\frac{1}{2}$.

For antiparticles, this is reversed: only right-handed antiparticles have weak charge. In other words, our universe is not left-right symmetrical—we can tell whether we are looking at weak interactions directly or looking at them in a mirror. This asymmetry is one of many mysteries a unified theory seeks to explain.

When physicists unified the weak force with electromagnetism to create the electroweak theory, they combined the SU(2) fiber with a U(1) circle. This circle is not the same as the electromagnetic one; it represents a precursor to electromagnetism known as the hypercharge force, with particles twisting around it according to their hypercharge, labeled $Y$. Inside the combined four-dimensional electroweak Lie group, the $W^3$ circles combine with the hypercharge circles to form a two-dimensional torus. This torus can be sliced in many ways, just as every person has their own idiosyncratic way to slice a bagel. The fibers of particles known as Higgs bosons twist around the electroweak Lie group and determine a particular set of circles, breaking the symmetry—like someone insisting there is only one true way to cut a bagel. The Higgs does not twist around these circles, which then correspond to the massless photon of electromagnetism.

Perpendicular to these circles are another set that should correspond to another particle, which the developers of electroweak theory called the Z boson. The fibers of the Higgs bosons twist around the circles of the Z boson, as well as the circles of the $W^+$ and $W^−$, making all three particles massive. Experimental physicists discovered the Z in 1973, vindicating the theory and demonstrating how geometric principles have real-world consequences.

A good way to see how the electroweak theory works is to plot the weak charges and hypercharges of all known particles [see box on next four pages]. Because mathematicians call charge “weight,” this plot is known as a weight diagram. In this diagram, all particles line up on equally spaced oblique lines, corresponding to their electric charges. Electric charge is thus a specific combination of weak charge and hypercharge, determined by the Higgs bosons. By experimentally measuring the strength of the weak force, physicists know that the angle of these lines, known as the weak mixing angle, is about 30 degrees. Explaining the value of this angle is one of the most tangible and immediate goals of a unified theory of physics.

**COLORFUL PHYSICS**

In the standard model, the strong nuclear force that binds quarks into atomic nuclei corresponds geometrically to an even larger Lie group, SU(3). The SU(3) fiber is an eight-dimensional internal space composed of eight sets of circles twisting around one another in an intricate pattern, producing interactions among eight kinds of photonlike particles called gluons on account of how they “glue” nuclei together. As complicated as this fiber shape is, we can break it into comprehensible pieces. Em-

---

**From Electromagnetism to Geometry**

The geometric view of nature follows naturally from the way the world around us works. The simplest and most familiar examples are the forces of electricity and magnetism. Electric sparks, magnetic attraction and laser light are different manifestations of the electric and magnetic fields that pervade space. In fact, physicists think that everything in the world—all the forces of nature and even all the particles of matter—arises from different kinds of fields. The behavior of these fields hints at an underlying geometric structure.
bedded within it is a torus formed by two sets of untwisted circles, corresponding to two generators, $g^3$ and $g^8$. The remaining six gluon generators twist around this torus, and their resulting $g^3$ and $g^8$ charges form a hexagon in the weight diagram.

The quark fibers twist around this SU(3) Lie group, their strong charges forming a triangle in the weight diagram. These quarks are whimsically labeled with three colors: red, green and blue. A collection of matter fibers forming a complete pattern, such as three quarks in a triangle, is called a representation of the Lie group. The colorful description of the strong interactions is known as the theory of quantum chromodynamics.

Together, quantum chromodynamics and the electroweak model make up the Standard Model of particle physics, with a Lie group formed by combining SU(3), SU(2) and U(1), as well as matter in several representations. This structure is described by a weight diagram with four charge axes, which may be projected down to two dimensions and plotted. This diagram displays the crown jewels of modern physics. Every allowed particle interaction of the Standard Model may be found on it.

The Standard Model is a great success. But it presents several puzzles: Why does nature use this combination of Lie groups? Why do these matter fibers exist? Why do the Higgs bosons exist? How is gravity included? And there are other mysteries we have not even touched on. Why does nature use this combination of Lie groups? Why do these matter fibers exist? Why do the Higgs bosons exist? Why is gravity included? And there are other mysteries we have not even touched on.

The world of elementary particles is a veritable menagerie. Particles come in two broad types, bosons (which transmit forces) and fermions (which constitute matter). Each fermion can come in several varieties: particle or antiparticle, left- or right-handed, spin up or down, and, for quarks, one of three colors. Every particle, identified by its charges, can be plotted in a weight diagram.

GRAND (BUT NOT FULL) UNIFICATION

Although the electroweak and strong forces can both be described using fiber weak and strong forces. Physicists have asked whether some single fiber encompasses both. Instead of different Lie groups for each force, there would be a single, larger Lie group for all. They have good evidence for this idea: all these forces become close in strength at very short distances, indicating they are aspects of a single force. A Grand Unified Theory would describe this force, reproduce the Standard Model and make testable predictions.

In this way, investigators are trying to reproduce the earlier successes of finding why the chemical elements line up in the periodic table, representing the structure of atoms. Once chemists had gleaned this structure, they began making predictions for what properties the elements should have and what new elements might await discovery. Likewise, particle physicists today are trying to find out why the weight diagram of the Standard Model has the pattern it does, and once they do, they will be able to make predictions for what properties the particles should have and what new particles might exist.

The first attempt at such a theory was proposed in 1973, by Howard Georgi and Sheldon Glashow [see “A Unified Theory of Elementary Particles and Forces,” by Howard Georgi; Scientific American, April 1981]. They found that the combined Lie group of the Standard Model fits snugly into the Lie group SU(5) as a subgroup. This SU(5) GUT made some distinctive predictions. First, fermions should have exactly the hypercharges that they do—a highly nontrivial success. Second, the weak mixing angle should be 38 degrees, in fair agreement with experiments. And finally, in addition to the 12 Standard Model bosons, there are 12 new force particles in SU(5), called X bosons.
It was the $X$ bosons that got the theory into trouble. These new particles would allow protons to decay into lighter particles, which they cannot do in the Standard Model. In impressive experiments, including the observation of 50,000 tons of water in a converted Japanese mine, the predicted proton decay was not seen. Thus, physicists have ruled out this theory.

Despite the SU(5) theory's failures, its successes suggest that theorists are generally on the right track. A related Grand Unified Theory, developed around the same time, is based on the Lie group Spin(10). It produces the same hypercharges and weak mixing angle as SU(5) and also predicts the existence of a new force, very similar to the weak force. This new “weaker” force, mediated by relatives of the weak-force bosons called $W^\pm$, $W^0$ and $W^3$, interacts with right-handed fermions, restoring left-right symmetry to the universe at short distances. Although this theory predicts an abundance of $X$ bosons—a full 30 of them—it also indicates that proton decay would occur at a lower rate than for the SU(5) theory. So the theory remains viable.

Drawn a certain way, the weight diagram for the Spin(10) GUT shows that particle charges align in four concentric circles—an unusually pretty pattern [see left panel on next page]. The balance evident in this diagram arises for a deep reason: the Spin(10) Lie group with its 45 bosons, along with its representations of 16 fermions and their 16 antifermions, are in fact all parts of a single Lie group, a special one known as the exceptional Lie group E6.

The exceptional groups play an exalted role in mathematics. Because there are only so many ways circles can twist around one another, there are only a handful of different kinds of Lie groups. Mathematicians completed their classification a century ago. We have already met two, SU and Spin, encountered quite often in physics. And among the Lie groups there are five exceptional cases that stand out: G2, F4, E6, E7 and E8. These Lie groups have especially intricate structures and deep connections to many areas of mathematics.

The fact that the bosons and fermions of Spin(10) and the Standard Model tightly fit the structure of E6, with its 78 generators, is remarkable. It provokes a radical thought. Up until now, physicists have thought of bosons and fermions as completely different. Bosons are parts of Lie group force fibers, and fermions are different kinds of fibers, twisting around the Lie groups. But what if bosons and fermions are parts of a single fiber? That is what the embedding of the Spin(10) GUT in E6 suggests. The structure of E6 includes both types of particles. In a radical unification of forces and matter, bosons and fermions can be combined as parts of a superconnection field.

Although several people have criticized this idea because it combines fermions and bosons in a way that at first appears fundamentally inconsistent, it relies on solid mathematics. And the curvature of this superconnection, describing the twisting of E6 over spacetime, succinctly describes the dynamics and interactions of bosons and fermions in the Standard Model. But E6 does not include the Higgs bosons or gravity.

**TAKING GRAVITY FOR A SPIN**

Albert Einstein originally described gravity as the curvature of spacetime. His mathematical machinery was state-of-the-art at the time, but researchers have gradually adopted a more modern, equivalent description of gravity based on a fiber bundle.

At every spacetime point, we can imagine three perpendicular rulers and a clock, called a frame of reference. Without the frame, spacetime would not be “spacetime” but just a four-dimensional
fabric with no sense of orientation or distance. As we move to different points in spacetime, there are different sets of rulers and clocks, related to our original frame by a rotation. This rotation can be an ordinary rotation in space or, because Einstein showed that space and time are unified, a rotation of space into time. How the frame rotates from point to point is determined by the spin connection, more commonly known as the gravitational field. The Lie group of possible rotations in three spaces and one time direction is Spin(1,3)—the Lie group of gravity. We feel the force of gravity because the gravitational spin connection field is rotating our frame as we move through time, attempting to steer us toward Earth’s center.

Just as particles have different kinds of charge describing how they interact with Standard Model forces, they have a type of charge describing how they behave within space. Consider what happens if we rotate a ruler in space by 360 degrees: it returns to its original state. This ruler—and the gravitational frame field—has spatial spin charge of +1 or -1. But if we rotate a fermion, such as an electron, in space by 360 degrees, it does not return to the same state it started in. To return it to its original state, we have to rotate it by 720 degrees. The fermion has spin charge of ±½.

Spin charge plays a role in gravity because gravity, through the frame and spin connection, is related to the geometry of spacetime. As we did for the other forces, we can make a weight diagram for gravity based on spin connection. A particle's spatial spin charge is its internal angular momentum, and its temporal spin charge is related to its motion through space. Fermions whose spatial spin and motion align, plotted in the upper right or lower left of the diagram, make a right-handed corkscrew as they travel through space. Fermions with opposite motion and spatial spin are left-handed.

What is strange is that spin charge also has an unexpected relevance to the weak nuclear force. Only left-handed particles and right-handed antiparticles have weak charge and interact with the weak force. The fact that the weak force is sensitive to spin charge suggests that gravity and the other forces, though outwardly dissimilar, in fact have a deep relationship.

**E Pluribus Unum**

Now it is just a matter of putting the pieces together. With gravity described by Spin(1,3) and the favored Grand Unified Theory based on Spin(10), it is natural to combine them using a single Lie group, Spin(11,3), yielding a Gravitational Grand Unified Theory—as introduced last year by Roberto Percacci of the International School for Advanced Studies in Trieste and Fabrizio Nesti of the University of Ferrara in Italy. It brings us close to a full Theory of Everything.

The Spin(11,3) Lie group allows for blocks of 64 fermions and, amazingly, predicts their spin, electroweak and strong charges perfectly. It also automatically includes a set of Higgs bosons and the gravitational frame; in fact, they are unified as “frame-Higgs” generators in Spin(11,3). The curvature of the Spin(11,3) fiber bundle correctly describes the dynamics of gravity, the other forces and the Higgs. It even includes a cosmological constant that explains cosmic dark energy. Everything falls into place.

Skeptics objected that such a theory should be impossible. It appears to violate a theorem in particle physics, the Coleman-Mandula theorem, which forbids combining gravity with the other forces in a single Lie group. But the theorem has an important loophole: it applies only when spacetime exists. In the Spin(11,3) theory (and in E8 theory), gravity is unified with the other forces only before the full Lie group symmetry is broken, and when that

---

**E6 Theory**. An even grander unification idea is that not just the forces of nature but also the particles of matter are all parts of a single fiber shape. In fact, a shape known as E6 fits the bill. But this structure does not include the Higgs bosons or gravity.

**Gravity**. The force of gravity has two charges: spin in space (ωs) and in time (ωt), related to rotation and linear motion. Particles in the upper left and lower right have a left-handed corkscrew motion through space.

**Standard Model plus Gravity**. Combining the diagrams of the Standard Model and of gravity produces a single diagram with all the known particle fields. This diagram also shows a “frame-Higgs” field that unifies the Higgs bosons with the gravitational frame of reference. Because only left-handed fermions have weak charge, the puzzle fits together in a specific way.
is true, spacetime does not yet exist. Our universe begins when the symmetry breaks: the frame-Higgs field becomes nonzero, singling out a specific direction in the unifying Lie group. At this instant, gravity becomes an independent force, and spacetime comes into existence with a bang. Thus, the theorem is always satisfied. The dawn of time was the breaking of perfect symmetry.

The weight diagram of the Spin(11,3) theory is finely patterned and balanced. Its symmetry, like that of the Spin(10) GUT, hints at deeper, exceptional mathematics. This elegant pattern of particles is part of what is perhaps the most beautiful structure in all of mathematics, the largest simple exceptional Lie group, E8. Just as E6 contains the structure of the Spin(10) Grand Unified Theory, with its 16 fermions, the E8 Lie group contains the structure of the Spin(11,3) Gravitational Grand Unified Theory, with its 64 Standard Model fermions, including their spins. In this way, gravity and the other known forces, the Higgs, and one generation of Standard Model fermions are all parts of the unified superconnection field of an E8 fiber bundle.

The E8 Lie group, with 248 generators, has a wonderfully intricate structure. In addition to gravity and the Standard Model particles, E8 includes $W'$, $Z'$ and $X$ bosons, a rich set of Higgs bosons, novel particles called mirror fermions, and axions—a cosmic dark matter candidate. Even more intriguing is a symmetry of E8 called triality. Using triality, the 64 generators of one generation of Standard Model fermions can be related to two other blocks of 64 generators. These three blocks might intermix to reproduce the three generations of known fermions.

**COLLIDING WITH REALITY**

In this way, the physical universe could emerge naturally from a mathematical structure without peer. The theory tells us what Higgs bosons are, how gravity and the other forces emerge from symmetry-breaking, why fermions exist with the spins and charges they have, and why all these particles interact as they do. In July those of us studying the theory held an exciting and productive workshop in Banff in Alberta, Canada, and we are planning a follow-up. Although this new theory continues to be promising, much work remains to be done. We need to figure out how three generations of fermions unfold, how they mix and interact with the Higgs to get their masses, and exactly how E8 theory works within the context of quantum theory.

If E8 theory is correct, it is likely the Large Hadron Collider will detect some of its predicted particles. If, on the other hand, the collider detects new particles that do not fit E8’s pattern, that could be a fatal blow for the theory. In either case, any particles that experimentalists uncover will take their place in a weight diagram, leading us toward some geometric structure at the heart of nature. And if the structure of the universe at the tiny scales of elementary particles does turn out to be described by E8, with its 248 sets of circles wrapping around one another in an exquisite pattern, twisting and dancing over spacetime in all possible ways, then we will have achieved a complete unification and have the satisfaction of knowing we live in an exceptionally beautiful universe.

---

To explore the Standard Model and proposed extensions such as E8 theory, visit the Elementary Particle Explorer at [http://deferentialgeometry.org/epe](http://deferentialgeometry.org/epe).

---

**MORE TO EXPLORE**


---

*Embedding within E8*. Studying the partially assembled puzzle of the Standard Model and gravity, we see that the charges of all particles fit in the pattern of what is arguably the most intricate structure known to mathematics, the exceptional Lie group E8. E8 also has exotic particles such as mirror fermions (smaller glyphs) and bosons that mediate hitherto unobserved forces.

*E8 Theory*. The embedding within E8 suggests that every fiber there is—every force, every known particle of matter and a clutch of additional particles that might account for cosmic dark matter—could be parts of this one exquisite shape. E8 even has a special symmetry called triality that relates its parts, which might explain why fermions come in three progressively heavier varieties called generations. E8 theory may be the long-sought Theory of Everything.